MATHEMATICS:

FACTS, LEGENDS & MYTHS, APPLICATIONS

Roland Glowinski, 2/18/2010

My goal is to show you the ***diversity*** of ***Mathematics*** by addressing the following issues:

- **What is Mathematics ?**
- **Relation with the other sciences**
- **The applications of Mathematics**
- **The sociology of mathematics (women in Mathematics in particular)**
- **Some Great Challenges in Mathematics**

1. What Mathematics is about ?

People have been doing ***Mathematics*** for several thousand years, going back to Babylon, ancient Egypt and Greece, pre-Columbian America, etc.… Nevertheless, there is a strong misconception about ***Mathematics*** in the population in general, typical comments one hears being: **" It's boring"**, **"I never understood it"**, **" It amounts proving that A = A (or 0 = 0), so what's the point ?"**, **"Mathematics is not for women"**, **"Computers have made Mathematics obsolete"** and more like those.

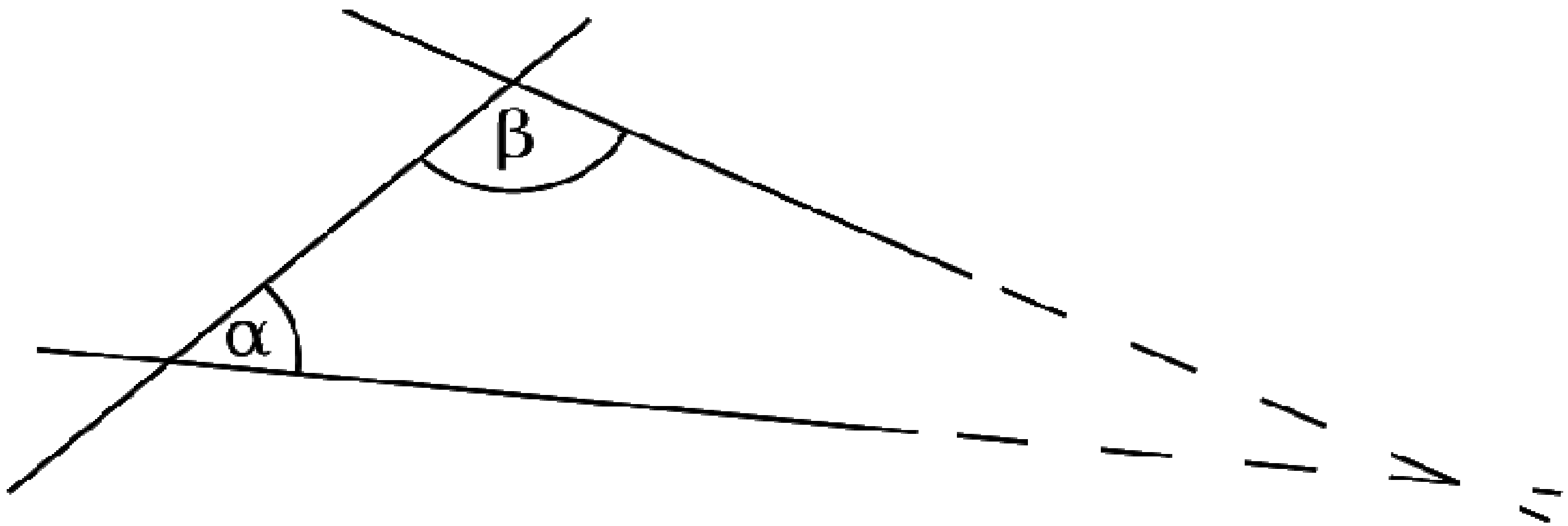
Although all of us have some idea of what is ***Mathematics***, let's try to define it:

According to ***WIKIPEDIA***, the main source of information these days:

- ***Mathematics*** is the study of ***quantity***, ***structure***, ***space*** and ***change***. ***Mathematicians*** seek out ***patterns***, formulate ***conjectures***, and establish truth by ***rigorous deduction*** from appropriately chosen ***axioms*** and ***definitions***.

In traditional ***logic***, an ***axiom*** or ***postulate*** is a proposition that is not proved or demonstrated but considered to be either ***self-evident***, or subject to ***necessary decision***.

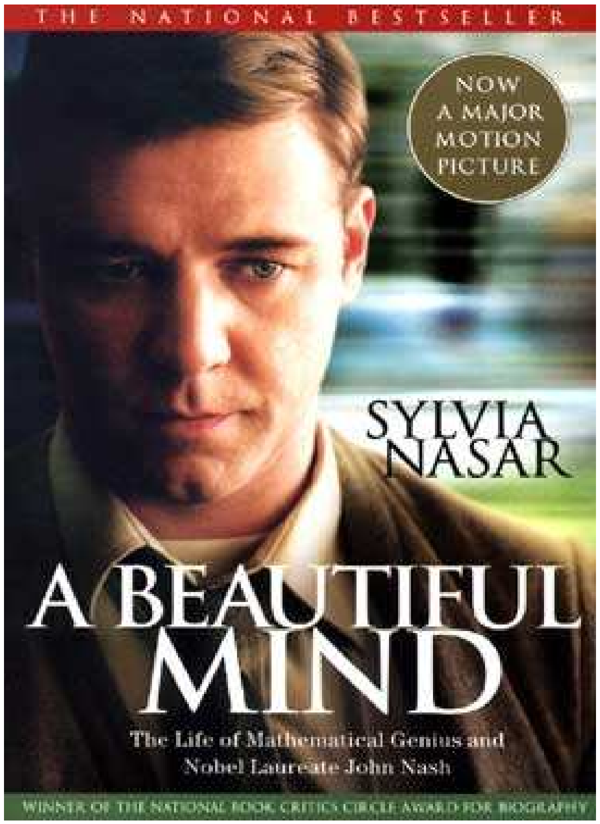
The parallel postulate: If two lines intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

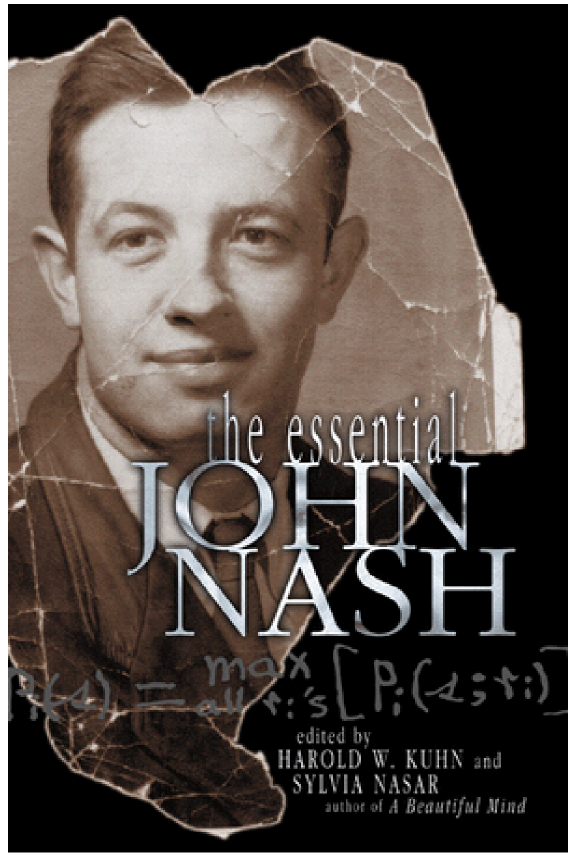


- ♦ Through the use of ***abstraction*** and ***logical reasoning***, mathematics evolved from ***counting, calculation***, ***measurement***, and the systematic study of the ***shapes*** and ***motions*** of physical objects. ***Practical mathematics*** has been a human activity for as far back as ***written records*** exist. Rigorous arguments first appeared in ***Greek mathematics***, most notably in ***Euclid's Elements***. Mathematics continued to develop, for example **in China in 300 BCE**, in **India in 100 CE** (it seems that the **zero** was (re) invented in **India** at that time**;** actually, the Mayas used also the zero before it was known in Europe), and in **Arabia in 800 CE**, until the ***Renaissance***, when mathematical innovations interacting with new ***scientific discoveries*** led to a rapid increase in the rate of mathematical discovery that continues to the present day.

Euclid (3rd century BC) viewed by ***Raphael***

- Mathematics is used throughout the world as an essential tool in many fields, including ***natural science***, ***engineering***, ***medicine***, and the ***social sciences***. ***Applied mathematics***, the branch of mathematics concerned with application of mathematical knowledge to other fields, inspires and makes use of new mathematical discoveries and sometimes leads to the development of entirely new mathematical disciplines, such as ***statistics*** and ***game theory*** (***J. Von Neumann***, ***J. Nash***). Mathematicians also engage in ***pure mathematics***, or mathematics for its own sake, without having any application in mind, although practical applications for what began as pure mathematics are often discovered (***Hilbert spaces***).


THE NATIONAL BESTSELLER
NOW A MAJOR MOTION PICTURE
SYLVIA NASAR
A BEAUTIFUL MIND
The Life of Mathematical Genius and Nobel Laureate John Nash
WINNER OF THE NATIONAL BOOK CRITICS CIRCLE AWARD FOR BIOGRAPHY


the essential JOHN NASH
edited by
HAROLD W. KUHN and
SYLVIA NASAR
author of A Beautiful Mind

2. A fundamental question

Is Mathematics a Science ?

The question is simple but the answer is not. The fact that for many years there was at ***Rice University*** (along side with the **Department of Mathema tics**) a department of ***Mathematical Sciences*** (now **Department of Computational and Applied Mathematics**) suggests that, it is indeed the case. However, for ***Richard Feynman*** (Physics Nobel Prize laureate in 1965), ***Mathematics*** is just a ***language*** plus a ***tool box***. If you do not find in the box the tool you need, you create (invent) one. It is what ***Feyman*** did with his ***path integral*** formulation of ***Quantum Mechanics***. But, doing so ***was Feynman acting as a Mathematician or as a Physicist ?*** The answer is not clear and the two above questions are fundamentally ***philosophical*** ones (implying that a meeting of the ***Houston Philosophical Society*** is the right place to formulate them).

Actually, for ***scientific giants*** like ***I. Newton***, ***L. Euler***, ***K.F. Gauss***, ***J. Fourier***, and very likely ***H. Poincaré*** and ***J. Von Neumann***, there was no clear boundary between ***Mathematics*** and ***Mechanics/Physics***. They thought of themselves as ***Natural Philosophers***. Indeed, ***H. Poincaré***, in addition to being a ***mathematician*** and a ***physicist*** was also a ***philosopher*** in the usual sense of the word (he was elected member of the ***American Philosophical Society*** in 1899). For ***H. Poincaré***, ***Mathematics*** is a science because its reasoning travels from the particular to the general (and conversely). A mathematician who conducts her/his mental experiments with sufficient rigor could derive the rules that govern the rest of the imaginary (?) terrain she/he shares with other mathematicians. In other words, when proving that **A = A** (or **A = B**), she/he has also explained what **A** and **B** are at some fundamental level and where other **A**'s can be found or how they can be constructed. For non-philosophers this may seem a bit abstract. An ***example*** may help.

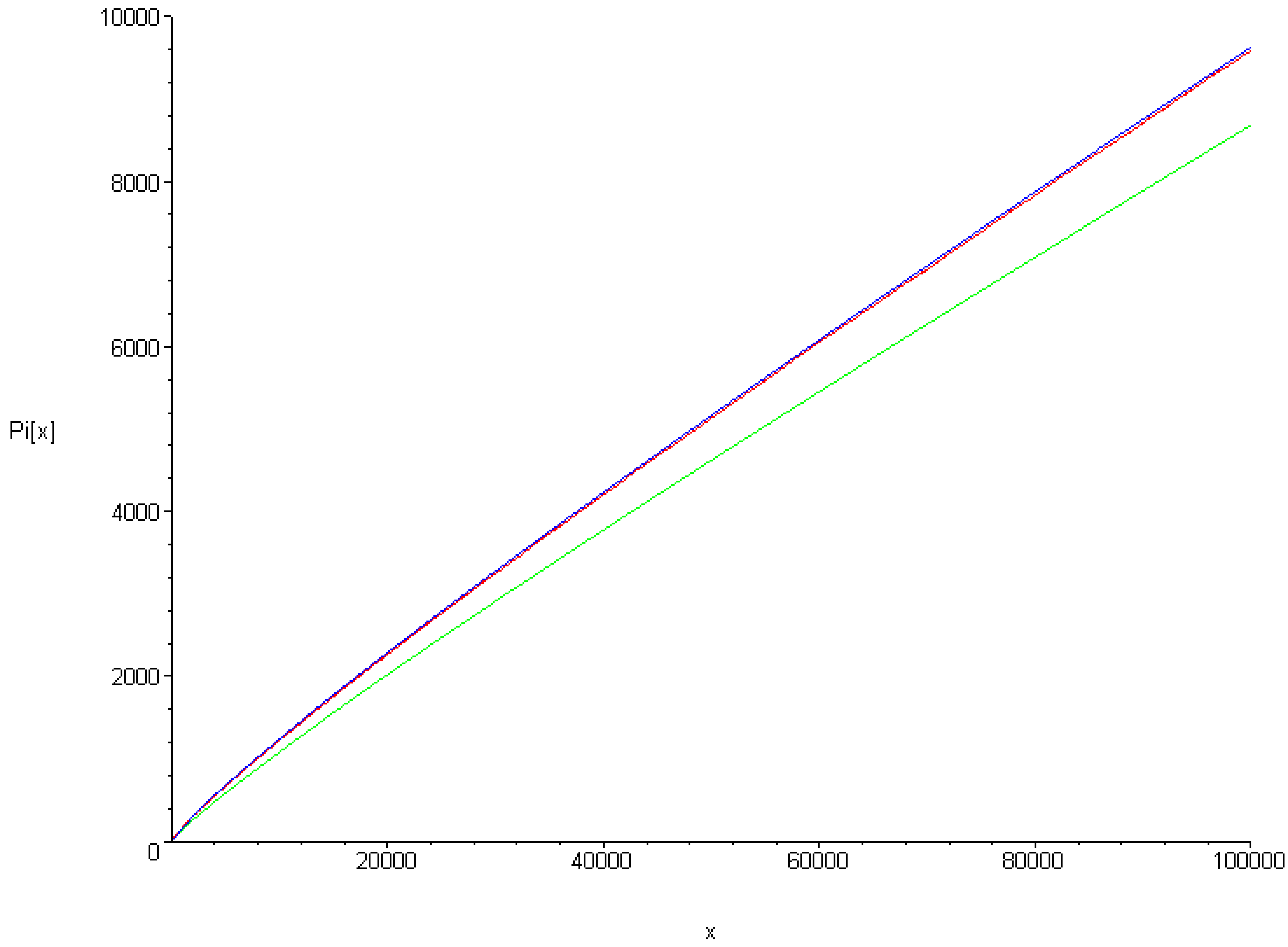
Henri Poincaré (1854-1912)

This example has to do with the ***prime numbers distribution*** and will show how ***mathematical reasoning*** and ***physical reasoning*** are close (and since ***Physics*** is a science…). You all know what is a ***prime number*** (**2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37,.., 97** and so on). Let us denote by ***π*(*n*)** the ***number of primes smaller or equal to n*** (**≤ *n***) (for example **$\pi(12) = 5$**). By analyzing ***tables*** containing several thousand of primes, ***K.F. Gauss*** (1777-1855) ***conjectured*** in **1796** (he was just ***nineteen*** !) that the ***ratio*** of $\pi(n)$ by $n/\log n$, that is $\pi(n)\log n/n$, ***is as close to 1 as one wants if one takes n large enough***. Using ***mathematical notation*** the ***prime number conjecture*** (and later ***theorem***) reads as:

The Prime Number Theorem

$$\lim_{n \to +\infty} \frac{\pi(n) \log n}{n} = 1$$

Top curve: $\pi(n)$ Lower curve: $n/\log n$



It took exactly ***hundred years*** of active research for the ***prime number conjecture*** to become the ***prime number theorem***. This took place in **1896** when independently, simultaneously and using closely related methods, ***Jacques Hadamard*** (France, 1865-1963) and ***Charles de la Vallée-Poussin*** (Belgium, 1866-1962) proved that ***Gauss*** was right. Their proofs are quite sophisticated and use the so-called ***Riemann zeta function*** defined by

The zeta function

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + ... + \frac{1}{n^s} + ... = \sum_{n=1}^{+\infty} \frac{1}{n^s}$$

Question: What the ***zeta function*** has to do with the ***prime numbers*** ?

Answer: The ***Euler's Golden Key***

$$\zeta(s) = \frac{1}{1-\frac{1}{2^s}} \times \frac{1}{1-\frac{1}{3^s}} \times \frac{1}{1-\frac{1}{5^s}} \times \frac{1}{1-\frac{1}{7^s}} \times \dots\times \frac{1}{1-\frac{1}{p^s}} \times \dots\dots$$

that is

$$\zeta(s) = \prod_{p \text{ prime}} \frac{1}{1-\frac{1}{p^s}}$$ (***Euler*** proved that $\zeta(2) = \frac{\pi^2}{6}$)

Simpler proofs of the ***Prime Number Theorem*** have been found during the 20th century but none is very simple.

We hope that this example has convinced you that indeed ***Mathematics is a science*** (or at least more than just a language): One (***Gauss***) started from ***observations*** (like in ***Physics***) to suggest a behavior of the prime numbers, then a sophisticated machinery was put together (in particular by ***B. Riemann***) to prove it. It is interesting to observe that most of the actors of this quite remarkable ***ma thematical adventure*** had strong interests in ***Mechanics*** and ***Physics*** (***Euler equations*** of ***Fluid Mechanics***). They were more than ***Mathematicians***, they were ***Natural Philosophers***.

Leonhard Euler (1707-1783)

K.F. GAUSS (1777-1855)

also known as the “***Prince of Mathematicians***”

Bernhard Riemann (1826-1866)

J. Hadamard (1865-1963), **Ch. de la Vallée-Poussin** (1866-1962)

Concerning the relations between ***Mathematics*** and ***Physics*** two of our favorite statements are:

“ ***S. Ulam*** went to ***Physics*** because he thought that he was not a good enough mathematician.
J. Von Neumann went to ***Physics*** because ***Mathematics*** was to easy for him.”

P.D. Lax (Abel Prize 2005)

“The unreasonable effectiveness of mathematics in the natural sciences"

Eugene Wigner (Physics Nobel Prize 1963)

commenting on the remarkable fact that even the "purest" mathematics often turns out to have practical applications.

Actually, it is very likely that ***Plato*** (428-348, BC) would not have been surprised by ***E. Wigner*** statement. Why ? Because ***Plato*** was believing that our world is the ***shadow*** (that is, mathematic ally a ***projection***) of a ***perfect world*** (***universe***), this perfect world being a ***mathematical*** entity governed by ***mathematical relations***. By ***meditation*** human beings are able to enter in communication with this perfect mathematical universe, resulting in ***mathematical discoveries*** (**not inventions**). These mathem atical facts, structures, etc…have been ***pre-existing*** their discovery by human beings. Some 1st class mathematicians and physicists are not far from believing that ***Plato*** may be right after all. Another consequence, is that our imperfect world being a projection of the perfect mathematical one, it is not surprising, that mathematics work pretty well to explain and describe what we call ***natural phenomena***.

3. Mathematics and Computers

Without Mathematics and Mathematicians there will be no Computers

(1) ***Mathematics*** are used to design ***semi-conductor*** materials and the ***electronic circuits*** made with these materials.

(2) In ***computers*** these electronic circuits operate according to ***very strict mathematical rules***

(3) ***Programming languages*** are created using ***mathematical tools***

(4) ***Google*** relies on very sophisticated ***mathematical tools***. So does ***MATLAB***

(5) ***Von Neumann*** (one of the greatest mathematicians of all time) played a fundamental role in the ***design of modern compu ters*** and ***MICROSOFT*** recruited ***M. Freedman*** (a ***Fields medalist*** for his contributions to the ***Poincaré conjecture***).

4. Applied and Computational Mathematics

If there is an area where ***Mathematics*** and ***Computers*** **coopera te** instead of **competing** it is at the solution of a large variety of problems from the ***natural***, ***engineering***, ***medical***, ***social***, ***economical***,***…sciences***: ***One needs the other***.

The approach is always the same:

(i) Using the ***laws*** of physics, chemistry, mechanics,….., one builds a ***mathematical model*** describing the system under consideration (***differential equations*** are common at that stage).

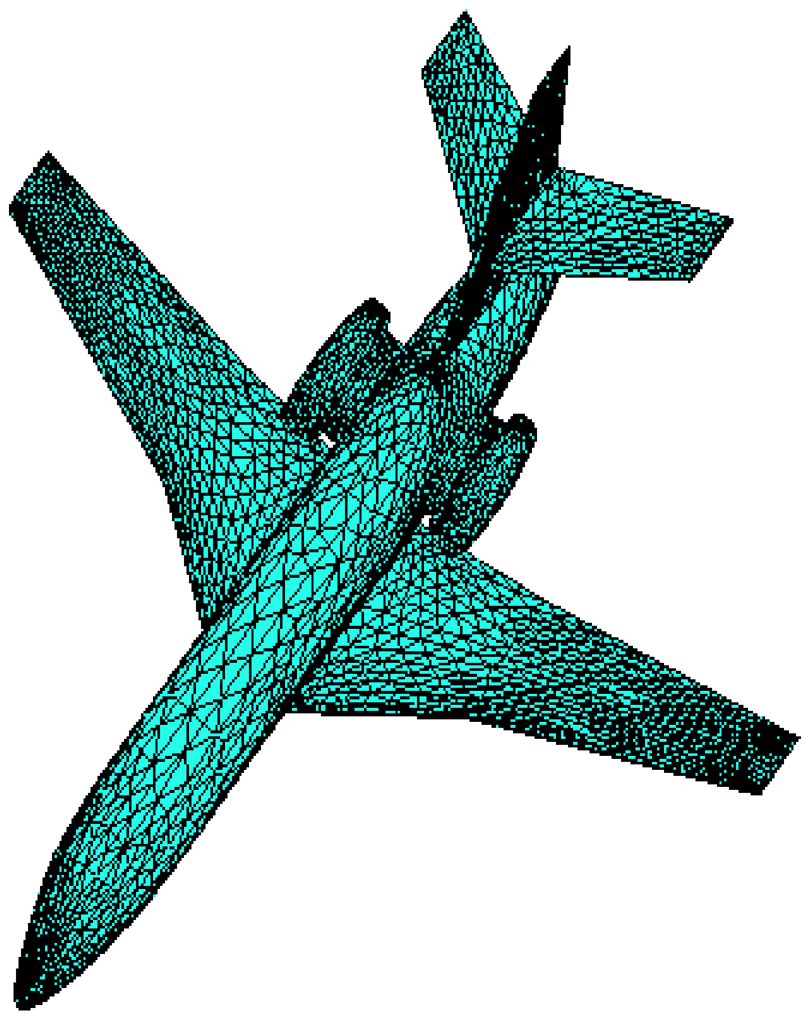
(ii) We ***approximate*** the above model by a simpler one the computer can treat via an ***appropriate programming***.

(iii) The results are ***validated*** by comparison with experimental data, exact solutions, results obtained by other methods, etc

The above approach has been used to design **cars**, **airplanes**, **rockets**, **boats**, **nuclear** and **chemical reactors**, **trips** to the **moon** and the **planets**, **GPS systems**, **cell phones**, **wind turbines**, **weapons** (nuclear and others), to **simulate sea currents** in the **Gulf of Mexico** and elsewhere, to **find oil & gas**, to **predict the weather**, to design **medical** and **dental prosthesis**, **new drugs**, to **manage companies**, **hedge funds**, **stock portfolios**, to design **codes** difficult to break down or contribute to breaking other people code (explaining why the ***NSA*** employs hundreds of Math PhD), to do **genome sequencing**. Focusing on ***Houston***, ***pure***, ***applied*** and ***computational*** mathematical activities take place at ***Rice University*** and ***University of Houston***, ***NASA***, the ***Medical Center*** (***MD Anderson*** in particular), the ***Oil & Gas*** companies, ***financial*** companies, ….. Some selected examples are:

It seems that Computational & Applied Mathematics can contribute to the building of a star

A two-jet engine airplane: The Falcon 50



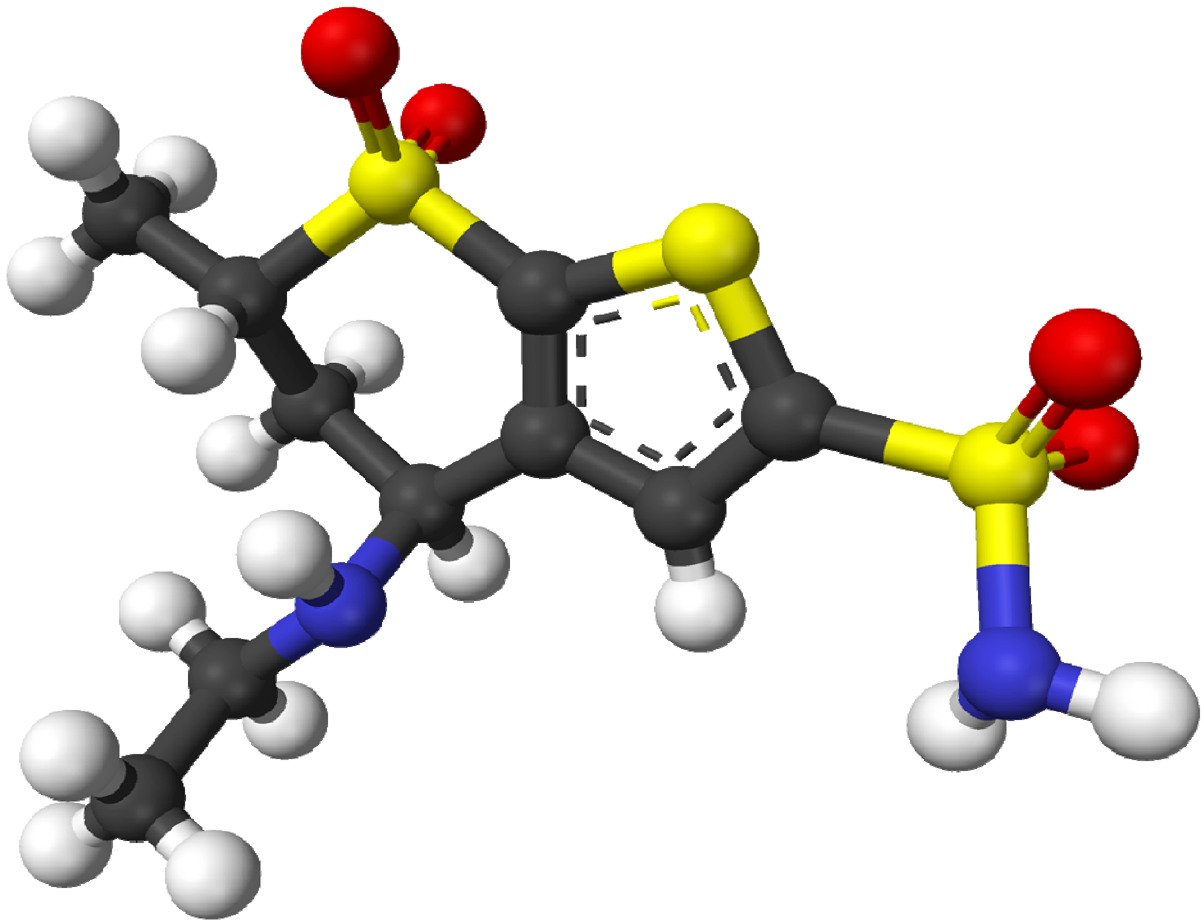
The ***Boeing 777*** is the **1st** ***fully computer designed airplane*** using in particular the software ***CATIA*** from ***Dassault System***. ***Catia*** was also used to design the ***W. Hobby-Eberly Telescope*** at the ***McDonald Observatory***, in Fort Davis, West Texas

W. Hobby-Eberly Telescope

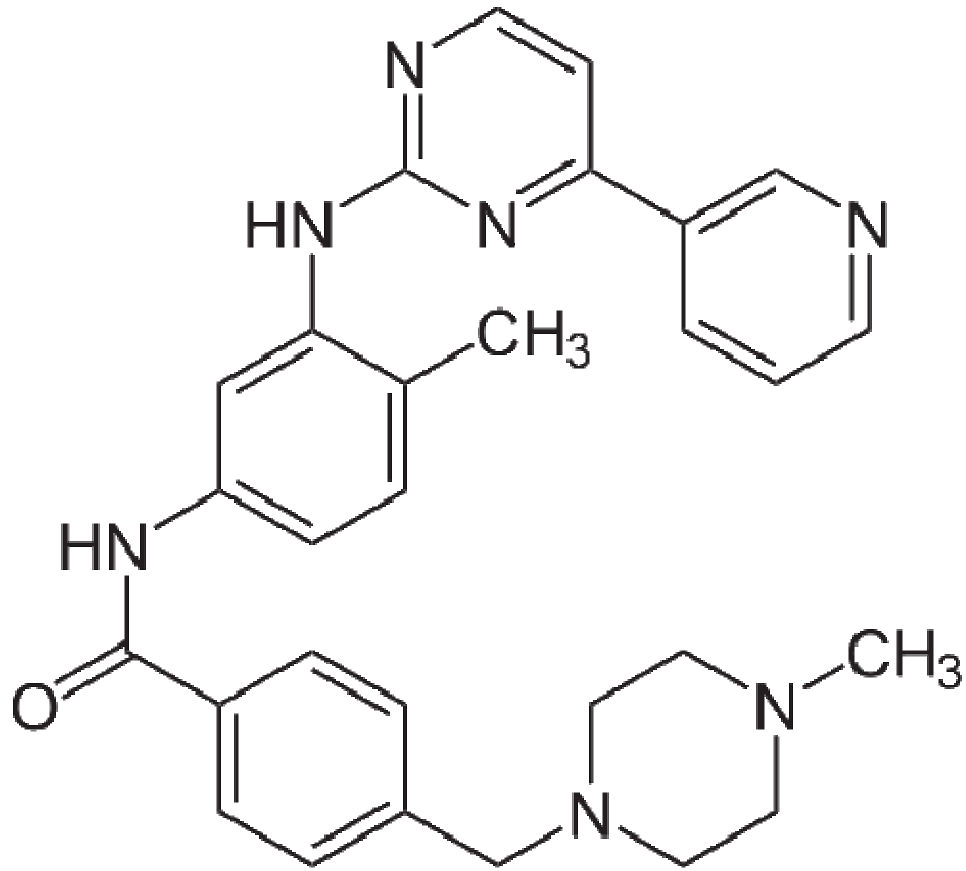
4th largest mirror telescope in the World

The **1st computer designed FDA approved drug**

is the **Dorzolamide**, used to treat **glaucoma**



Computer Designed FDA Approved Anti-Cancer Drug Imatinib/Gleevec



Imatinib bound to Abl-Kinase protein

5. Nobel & Abel Prizes, Fields Medals

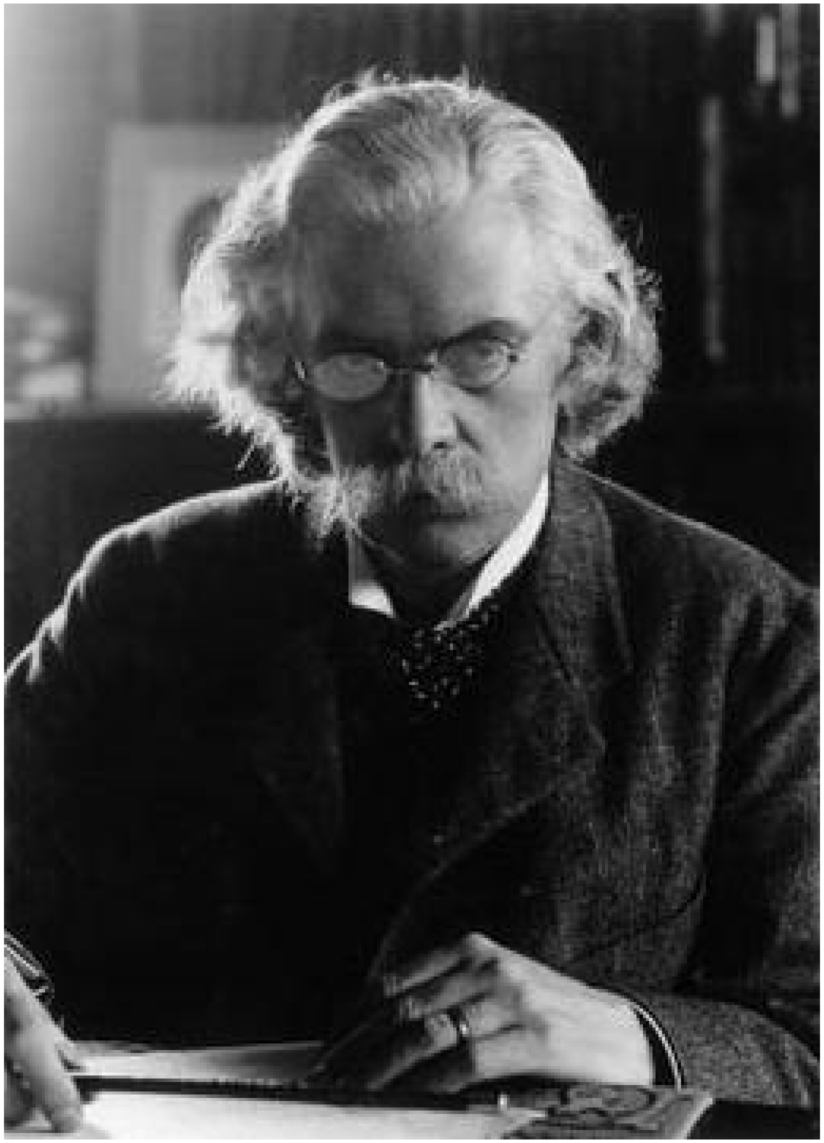
A question asked quite often is the following one:

Why there is no Nobel Prize in Mathematics ?

The traditional answer is that ***Alfred Nobel***'s wife or mistress had an affair with a mathematician. According to various sources, this is not a good explanation. In **2005**, while participating at a confer ence in ***Stockholm*** I heard a more convincing explanation:

The reason for which there is no ***Nobel Prize*** in ***Mathematics*** is

Magnus Gustav (Gösta) Mittag-Leffler (1846-1927)



The story as told us in June 2005 by the director of the ***Mittag-Leffler Institute*** in ***Stockholm*** looks like this (**short version**):

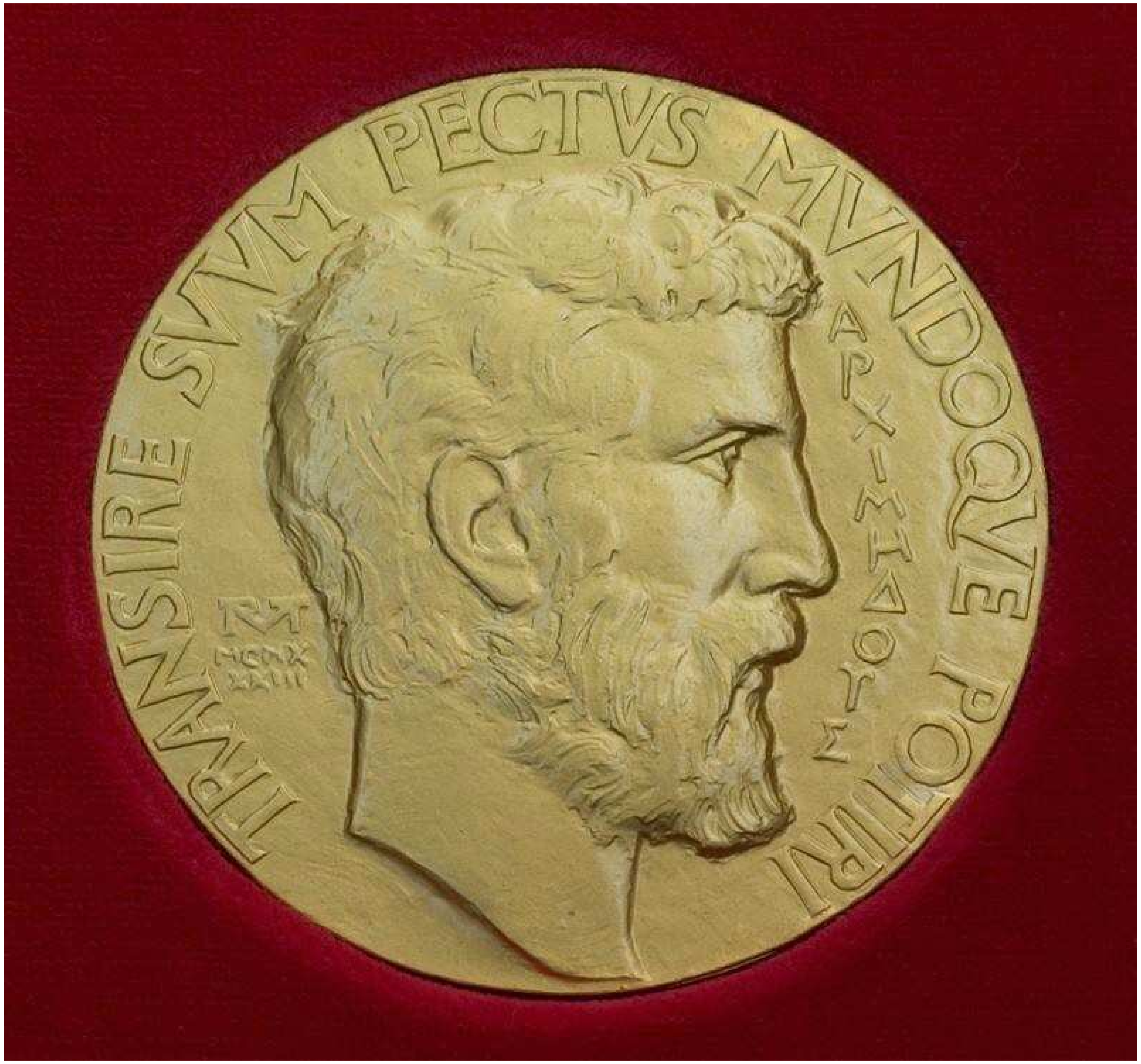
- When the ***Nobel Prizes*** were created (around **1900**) the leading Swedish mathematician was ***G. Mittag-Leffler***.

- ***Alfred Nobel*** strongly disliked ***Mittag-Leffler*** for he thought that he was vain, arrogant and an operator (not in the mathematical sense).

- The problem for ***A. Nobel*** was that ***Mittag-Leffler*** was an excellent mathematician who could have won the ***Nobel Prize*** on his own merit. This would have been too much for ***A. Nobel*** who decided therefore that ***there will be no Nobel Prize in Mathematics***.

Since there was no ***Nobel Prize in Mathematics***, there was room for substitutes. From **1936** to **2002**, this role was played by the ***Fields Medals*** given **every four years** to mathematicians **39 years old at most** the year before the year they will receive the reward (not much money-compared to Nobel Prizes-is attached to the medal: C$ 15,000 in 2006). From **1936** to **2006**, **48** mathemati cians received a ***Fields Medal***, among them **11** from the **USA**, **8** from **France** and **7** from **Russia/Soviet Union** and from the **UK**. So far, the ***Fields Medal*** has never been awarded to a woman mathematician.

The Fields Medal



The limits of the ***Fields Medal*** concept appeared clearly in the **mid-nineties** when **Andrew Wiles** (***President Lecture*** at **Rice U** in **2008**) completed the proof of the ***Fermat Last Theorem*** (formu lated in **1637**). When **A. Wiles** put the final touch to his proof, he was over 40 (by not much) and therefore not eligible for a ***Fields* Medal**. **A. Wiles** received many awards for his work, but none of the caliber of a **Fields Medal**. Fortunately, since **2003**, mathema ticians can be awarded (by the ***Royal Academy of Norway***, like the ***Peace Nobel Prize***) the ***Abel Prize*** (**US$ 1 million**). So far, the ***Abel Prize*** has been awarded to **9 mathematicians** (**4** being from the ***US***), none being a woman; it is very likely that ***A. Wiles*** will get the ***Abel Prize*** in a near future, since he definitely deserves it for completing a 3.5 century quest at the origin of many progresses in Mathematics.

My insistence on the absence of women in the ***Fields Medals*** and ***Abel Prize*** laureates is not derogatory; ***quite the opposite !***

6. Women in Mathematics

The **1st** ***famous*** **woman mathematician** was ***Hypatia of Alexandria*** (370-415). She was also a ***philosopher*** and a ***astronomer*****.** A ***neoplatonist*** philosopher and teacher, living in ***Roman Egypt***, she was murdered by Christians who felt threatened by her ***scholarship***, ***learning***, and ***depth of scientific knowledge.***

Hypatia of Alexandria (370-415)(by ***Raphael***).

The first notable ***French woman mathematician*** was ***Gabrielle, Emilie, Marquise du Châtelet*** (**1706-1749**). Actually she was also a very distinguished ***physicist*** who translated ***Newton's Principia Mathematica*** from Latin to French, adding her own mathematical commen ts to her translation. In fact she was the **1st** to suggest that the ***energy of a moving body*** is proportional to MV^2 and not **MV**, as others (including ***Newton***) were believing.***Voltaire*** (one of her lovers) was saying of her that she was:

"a great man whose only fault was being a woman"

Gabrielle, Emilie, Marquise du Châtelet (1706-1749)

Due to time limitation, I will mention only briefly ***Sophie Germain*** (France, 1776-1831), known for her contributions to ***Fermat Last Theorem*** (she was also a ***physicist*** and a ***philosopher***)and ***Ada King*** (***Lord Byron***' daughter), ***Countess of Lovelace*** (England, 1815-1852). ***Ada Byron*** assisted ***Charles Babbage*** in the design of a ***mechanical computer*** (the ***Analytical Machine***) and is credited for writing the **1st** ***computer program*** in History. As a tribute to her, the name ***ADA*** was given to a ***computer language*** developed for the ***US DOD*** in the eighties.

Sophie Germain

1776-1831 (by **Leray**)

Ada Byron

1815-1852

The fact of being a woman was definitely an handicap for the woman mathematicians we mentioned; they did not get the recognition and scientific positions that their talent was justifying. The same comments apply to ***Sofia/Sonya Kovalevskaya*** (Russia/Sweden, 1850-1891) and ***Emmy Noether*** (Germany/USA,1882-1935). In addition to being outstanding mathematicians whose results are still used today, they had very significant contributions to ***Physics***. ***Emmy Noether*** became in **1932** the **1st** ***woman*** to give a ***plenary lecture*** at the ***Mathematical Congress of Mathematicians*** (created in **1897** and held every four years); the **2nd** woman mathematician to repeat that feat is ***Karen Uhlenbeck*** (**UT Austin**), in **1990**. Since then, the situation has significantly improved. By the way ***several movies*** have been dedicated to the life of ***S. Kovalevskaya.*** Another commonality between these woman mathematicians is that they all died fairly young.

Sofia Kovalevskaya

1850-1891

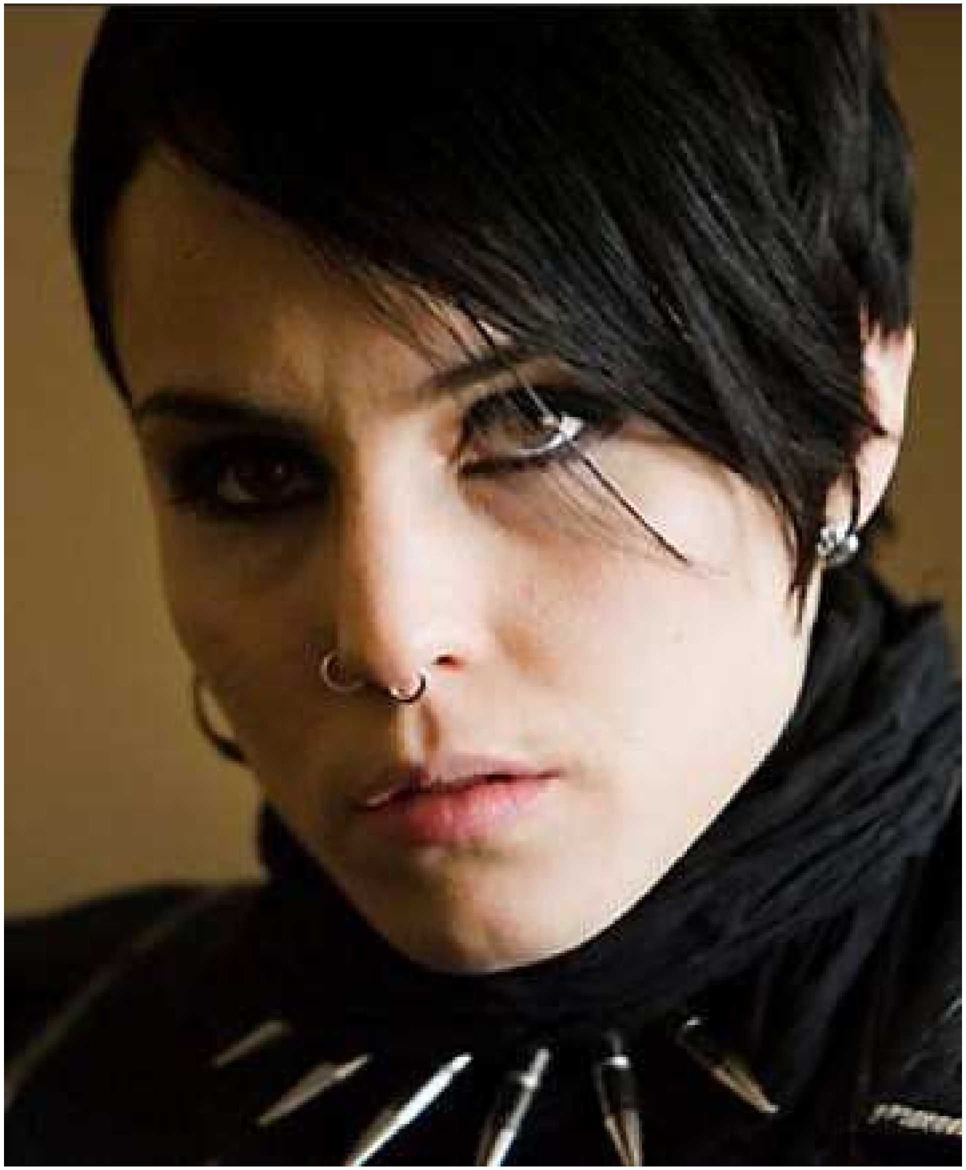
Emmy Noether

1882-1935

Talking of books and movies,

my *favorite woman mathematician* is

Lisbeth Salander (Sweden)



In **Volume 2** (***The Girl Who Played with Fire***) of the ***Millennium*** saga by ***Stieg Larsson*** (**1954 – 2004**) she found a simple proof (typically one page size) of the ***Fermat Last Theorem***, but had no time to write it. Unfortunately she got a head injury while fighting (very) bad guys. Since ***Stieg Larsson*** died after completing **Volume 3**, where our heroine realizes she does not remember the proof, we have to rely on **A. Wiles** for the only proof known so far.
Several conclusions can be drawn:

(i) Some men strongly believe in the scientific capabilities of women (as did ***Mittag-Leffler***, ***Einstein***, ***Hilbert***).

(ii) ***Mathematics*** has penetrated the ***general literature***.

(iii) It is time to have a “serious” discussion about ***Fermat Last Theorem***, since we mentioned it quite a few times.

7. Fermat Last Theorem and other Great Challenges in Mathematics

In 1637, ***Pierre de Fermat*** (1605(?)-1665), a ***French lawyer*** and ***amateur mathematician***, claimed that he had a short proof of the fact that if n is a ***positive integer larger than 2*** ($n > 2$), there ***are no positive integers*** a, b and c such that

$$a^n + b^n = c^n$$

(we have $3^2 + 4^2 = 5^2$). It took ***358 years*** to prove this result (**A. Wiles**) and ***Wiles*** proof is extremely complicated. Attempts at proving the **FLT** contributed greatly to progress in many areas of ***Mathematics*** and even ***Physics***. It is likely that ***Fermat*** never proved his Last Theorem. The story of the proof has generated plays, TV movies, several books and, beside **Stieg Larsson's**, several novels, such as ***Le Théorème du Perroquet*** (***The Parrot Theorem***).

A recently solved great challenge is the ***Poincaré Conjecture*** (**PC**) formulated by ***H. Poincaré*** in **1904**. It concerns some properties of the ***multidimensional sphere***

$$x_1^2 + x_2^2 + \dots\dots\dots\dots + x_n^2 = R^2$$

when $n > 3$, the ***most difficult case*** being $n = 4$. By the beginning of the **1980's**, the **PC** has been proved for all $n > 4$. It took **20** more years to prove it for $n = 4$, this feat being achieved by ***Gregory Perelman*** (Russia), relying on an approach suggested by ***Richard Hamiton*** (Columbia U). The final proof of the **PC** has generated various controversies, several books and even a very long article in the ***New Yorker*** magazine

Manifold Destiny

A legendary problem and the battle over who solved it

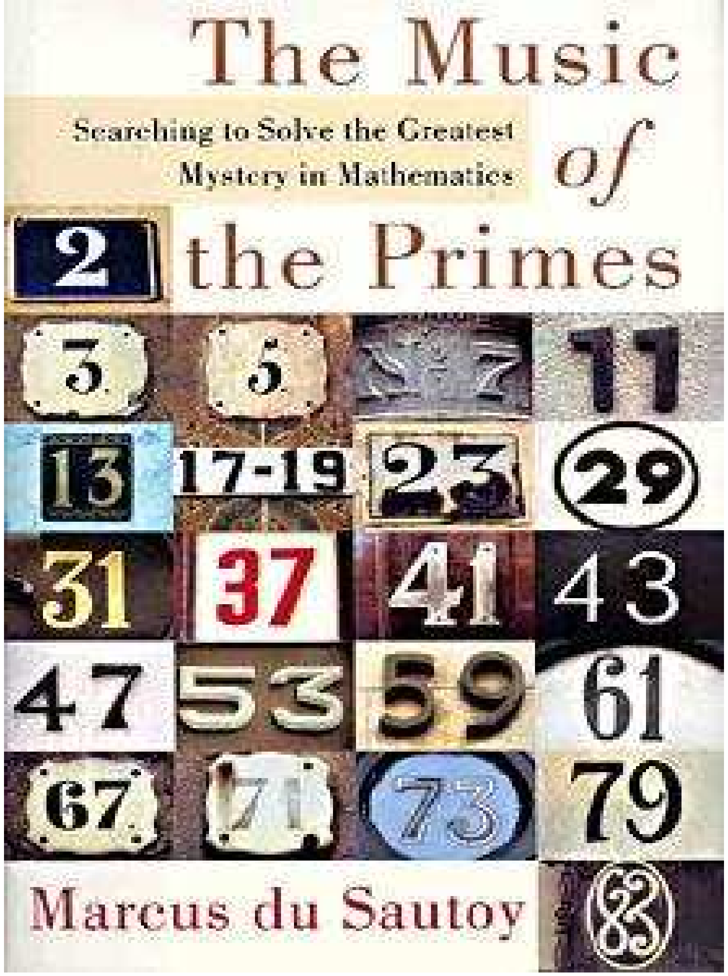
by Sylvia Nasar and David Gruber

August 28, 2006

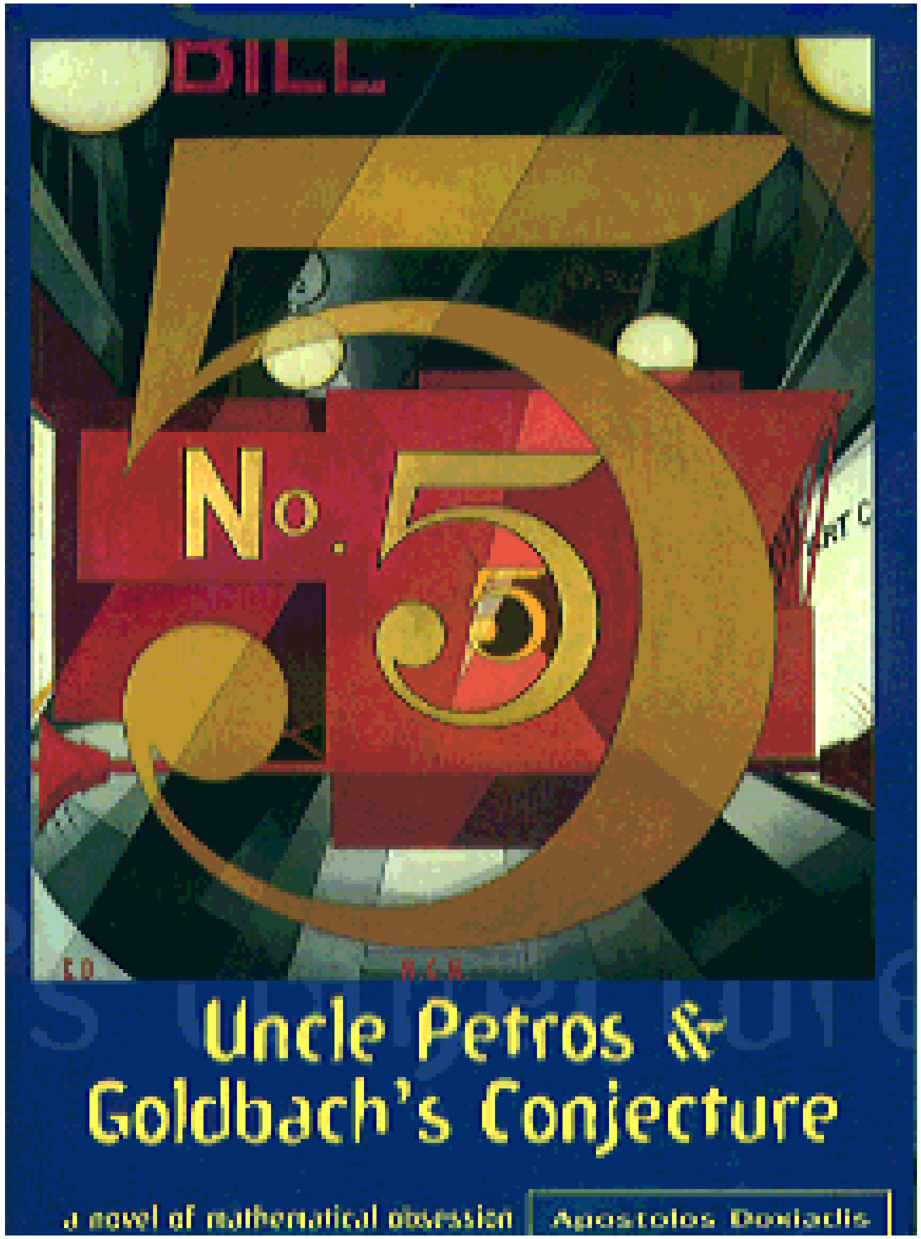
Two celebrated ***unsolved problems*** in ***Mathematics*** are the ***Riemann Hypothesis*** (**RH**) and the ***Goldbach Conjecture*** (**GC**).

Since the proof of the **FLT** the **RH** (**Riemann, 1859**) is now the most celebrated ***unsolved problem*** in Mathematics. It concerns some fine property of the ***Zeta function*** and its solution will improve our understanding of prime ***numbers distribution*** much beyond the ***Prime Number Theorem***.

The **RH** has attracted the attention not only of mathematicians but also of ***Physicists***, such as ***Sir Michael Berry*** (FRS). It as also motivated several books, among them



From my point of view, the ***Goldbach conjecture*** (**Goldbach**, **1742**) is deeper than the **FLT** and even easier to formulate. It states that: ***any*** **even integer** ***larger than 2 is the sum of two primes*** (**52 = 5 + 47 = 11 + 41**, for example); actually, using ***computers***, the **GC** has been **verified up to** 10^{18}. The **GC** has motivated, among other products, a TV movie and the fun reading book (by **Apostolos Doxiadis**)


BILL
No. 5
Uncle Petros &
Goldbach's Conjecture
a novel of mathematical obsession
Apostolos Doxiadis

Our insistence on challenging problems such as the ***FLT***, ***PC***, ***RH***, ***GC*** has to do with the fact that in order to solve them, ***mathematicians*** have developed ***tools*** and ***methods*** which found (and still find) ***applications*** to the solution of other problems, including applied ones.

8. Two mathematical tragedies

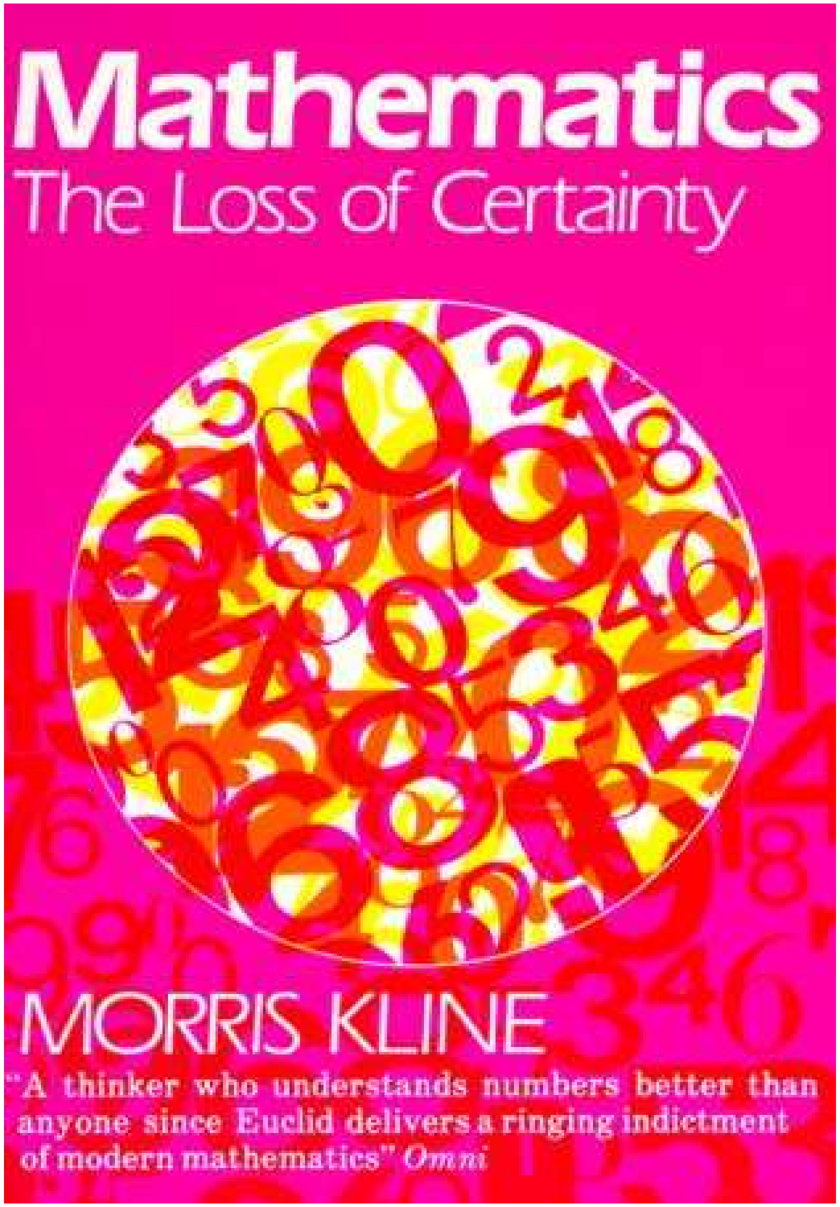
The **1st** one took place in ancient Greece/Italy, around **500 BC**. At that time ***Pythagoras*** (of Samos, **569-475 BC**) founded a ***philosophical*** and ***religious*** school in ***Croton*** (south of modern Italy). ***Mathematics*** was playing a fundamental role in Pythagoras philosophy, in particular the belief that numbers are either ***integers*** or the ***ratio of 2 such integers*** (what we call ***rational*** numbers today). A ***legend*** says that when one of the School members came with a proof that $\sqrt{2}$ (1. 414….) ***could not be rational***, School members killed him by throwing him away in the sea from a cliff (***if you don't like the message……***); as you can see some people were taking Mathematics ***very seriously*** at that time.

The **2nd** (kind of) **tragedy** took place more recently: in the early **1930's**, ***Kurt Gödel*** (Austria/USA, 1906-1978) proved that

Any consistent mathematical system which includes arithmetic contains necessarily true theorems which can not be proved using the rules at the basis of the system

This result, known as the ***Gödel Incompleteness Theorem*** was perceived as a ***scientific earthquake*** since it was suggesting that the ***foundations of Mathematics*** are not as firm as believed (and since ***Physics*** rely on ***Mathematics*** in some essential way…). This took place 80 years ago; nothing bad has happened so far, ***mathematics are used everywhere*** today and are a most remarkable success story.

If you want to know more about Gödel Incompletenes Theorem and the history of Mathematics read



My conclusion will be that

Math is cool

If you are not convinced

WALL STREET JOURNAL, **JANUARY 26, 2009, 11:20 A.M. ET**

CAREERS

Doing the Math to Find the Good Jobs

Mathematicians Land Top Spot in New Ranking of Best and Worst Occupations in the U.S.